FY07 Parameters for Injection, Acceleration, and Extraction of Gold Ions in Booster, AGS, and RHIC

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July 9, 2007

The Tables in this note contain the nominal FY07 parameter values for the injection, acceleration, and extraction of Gold ions in Booster, AGS, and RHIC. The ions in Booster, AGS, and RHIC are Au^{31+}, Au^{77+}, and Au^{79+} respectively.

1 Basic Parameters

A Gold ion with charge eQ has $N = 197$ Nucleons, $Z = 79$ Protons, and $(Z - Q)$ electrons. (Here Q is an integer and e is the charge of a single proton.) The mass is

$$m = au - Qm_e + E_b/c^2 \tag{1}$$

where $a = 196.966552$ is the atomic mass [1, 2] of the neutral Gold atom, $u = 931.494013$ MeV/c^2 is the unified atomic mass unit [3], and $m_e = .510998902$ MeV/c^2 is the electron mass [3]. E_b is the binding energy of the Q electrons removed from the neutral Gold atom. This amounts to 0.327 MeV for the fully stripped Gold ion as calculated by Trbojevic [4].

In a circular accelerator the ion moves along an orbit of circumference $2\pi R$ with revolution frequency f. The radius of the orbit is R. The velocity of the ion is then

$$v = 2\pi R f. \tag{2}$$

This gives momentum, energy, and kinetic energy

$$p = mc\beta\gamma, \quad E = mc^2\gamma = \sqrt{p^2c^2 + m^2c^4}, \quad W = E - mc^2 \tag{3}$$

where

$$\beta = v/c, \quad \gamma = 1/\sqrt{1 - \beta^2}. \tag{4}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = kp/Q \tag{5}$$

where $k = 10^9/299792458$ and p is the momentum in units of GeV/c. The angular frequency is

$$\omega = 2\pi f. \tag{6}$$

We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{7}$$

where γ_t is the transition gamma.

2 RF Parameters

The stationary bucket area is

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2} \tag{8}$$

where h is the RF harmonic number, V_g is the total RF gap voltage per turn, and the subscript "s" denotes parameter values for the synchronous particle.

The half-height of a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \left|(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s\right|^{1/2} \tag{9}$$

where ϕ_s is the synchronous phase.

The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{10}$$

where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

The width of a bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{11}$$

where the phase ϕ_e satisfies

$$\cos(\pi - \phi_s) - \cos\phi_e = -(\pi - \phi_s - \phi_e)\sin\phi_s. \tag{12}$$

The area of a bucket is

$$A_{\text{bk}} = \alpha(\phi_s)A_S \tag{13}$$

where

$$\alpha(\phi_s) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} |(\pi - \phi_s - \phi)\sin\phi_s - \cos\phi_s - \cos\phi|^{1/2}\,d\phi. \tag{14}$$

Below transition we have $\phi_e < \pi - \phi_s$ and the limits of integration are $\phi_L = \phi_e$ and $\phi_R = \pi - \phi_s$. Above transition we have $\pi - \phi_s < \phi_e$ and the limits of integration are $\phi_L = \pi - \phi_s$ and $\phi_R = \phi_e$. The integral $\alpha(\phi_s)$ must be evaluated numerically. An approximate expression is [5]

$$\alpha(\phi_s) \approx \frac{1 - \sin\phi_s}{1 + \sin\phi_s}. \tag{15}$$

The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s}\left\{\frac{-h\eta_s eQV_g\cos\phi_s}{2\pi E_s}\right\}^{1/2} \tag{16}$$

and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$. Note that measurement of F_s gives a value for $V_g\cos\phi_s$, while measurement of dB/dt gives a value for $V_g\sin\phi_s$. These can be used to obtain V_g and ϕ_s.

Let ϕ_l and ϕ_r be the phases at the left and right boundries of a bunch matched to a bucket. We have

$$\phi_l < \phi_s < \phi_r \tag{17}$$

and the width of the bunch is

$$\Delta t = \frac{\Delta\phi}{h\omega_s}, \quad \Delta\phi = \phi_r - \phi_l. \tag{18}$$

In terms of $\Delta\phi$ and ϕ_s we have

$$\phi_r = \frac{\Delta\phi}{2} + \arcsin\left\{\frac{\Delta\phi\sin\phi_s}{2\sin(\Delta\phi/2)}\right\} \tag{19}$$

and

$$\phi_l = -\frac{\Delta\phi}{2} + \arcsin\left\{\frac{\Delta\phi\sin\phi_s}{2\sin(\Delta\phi/2)}\right\}. \tag{20}$$

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Note that if $\Delta\phi$ is small we have

$$\sin(\Delta\phi/2) \approx \frac{\Delta\phi}{2}, \quad \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \approx \sin\phi_s \tag{21}$$

and

$$\phi_l \approx \phi_s - \frac{\Delta\phi}{2}, \quad \phi_r \approx \phi_s + \frac{\Delta\phi}{2}. \tag{22}$$

The half-height of the bunch is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \left|\cos\phi_r - \cos\phi_s + (\phi_r - \phi_s)\sin\phi_s\right|^{1/2}. \tag{23}$$

The area of the bunch is

$$A_{\mathrm{b}} = F(\phi_s, \Delta\phi)\, A_S \tag{24}$$

where

$$F(\phi_s, \Delta\phi) = \frac{\sqrt{2}}{8} \int_{\phi_l}^{\phi_r} \left|\cos\phi_l - \cos\phi + (\phi_l - \phi)\sin\phi_s\right|^{1/2} d\phi. \tag{25}$$

The integral $F(\phi_s, \Delta\phi)$ must be evaluated numerically. If $\Delta\phi$ is small we have

$$F(\phi_s, \Delta\phi) \approx \frac{\pi}{64}(\Delta\phi)^2 \left|\cos\phi_s\right|^{1/2}. \tag{26}$$

3   Lattice Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	C_r	m
C_E	$C_a/4$	$4C_r/19$	C_r	m
ρ	13.8656	85.378351	242.7806	m
γ_t	4.806	8.5	22.89	
Q_H, Q_V	4.757, 4.777	8.78, 8.72	28.19, 29.18	
Max β_H, β_V	13.5, 13.2	22.3, 22.2	48.6, 47.4	m
Max D_H	2.90	2.17	1.81	m

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{27}$$

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meters. Note that $4C_r/19 = 2\pi(128.45798)$ which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [6, 7]. The radius of curvature ρ in the lattice dipoles is given in Refs. [6, 7, 8, 9]. The other Booster and AGS lattice parameters were obtained from MAD runs. The RHIC lattice parameters are taken from Ref. [9] and from MAD runs by Steve Tepikian. (The maximum β_H, β_V, D_H listed for RHIC are the maxima in the arcs.)

4 Assumptions

The parameters values listed in Sections 5–7 are calculated assuming that:

1. The magnetic rigidity of the Au^{31+} ions at Booster injection is
$B\rho = 0.8813444$ Tm.

2. The magnetic rigidity of the Au^{31+} ions at Booster extraction is
$B\rho = 9.4307359$ Tm.

3. The magnetic rigidity of the Au^{77+} ions at AGS injection is
$B\rho = 3.7474454$ Tm.

4. The magnetic rigidity of the Au^{79+} ion at RHIC injection is the same as that of a proton with γ_p such that $G\gamma_p = 46.5$. Here $G + 1 = 2.792847337(29)$ and the proton mass is $m_p = 0.938271998(38)$ GeV/c^2 as reported in Ref. [10]. Thus $\gamma_p = 25.93639684$ and the proton momentum and energy are $P_p = m_p c \sqrt{\gamma_p^2 - 1} = 24.3173002$ GeV/c and $E_p = m_p c^2 \gamma_p = 24.3353949$ GeV. The rigidity is then $B\rho = kP_p = 81.1137824$ Tm.

5. The energy of the Au^{79+} ion at RHIC Store is 100 GeV per nucleon.

Please note that more digits are given for some parameters in Sections 5–7 than would be warranted by the precision with which the parameters could be measured; this is done for computational convenience. The notation "/N" in the Units column of the tables means "per nucleon".

5 Gold Parameters in Booster

Parameter	Injection	Extraction	Unit
Q	31	31	
m	183.457323	183.457323	GeV/c^2
W	182.75732/197	100.81632	MeV/N
cp	41.577830	444.89933	MeV/N
E	0.93218315	1.0320718	GeV/N
$B\rho$	0.8813444	9.4307359	Tm
β	0.044602641	0.43107403	
$\gamma - 1$	0.99618438/1000	0.10825850	
η	-0.955	-0.771	
ϵ_H (95%)	8.3π	8.3π	mm mrad
ϵ_V (95%)	3.9π	3.9π	mm mrad
h	6	6	
hf	0.39760735	3.8429173	MHz
R	201.780/(2π)	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during multi-turn injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parameter	Injection	Extraction	Unit
V_g	0.5	30	kV
A_S	2.540	23.04	eV s
dB/dt	0	80.0	G/ms
ϕ_s	0	48.25	degrees
F_s	0.412	2.225	kHz
A_{bk}	2.540	3.111	eV s
A_b	0.7263	1.499	eV s
Δt	1008	55.0	ns
ΔE	0.467	17.7	MeV

Parameter	Injection	Extraction	Unit
No. of Bunches	6	6	
Bunch Spacing	2515.044	260.2190	ns
Ions/Bunch	3.47/6	2.98/6	10^9
Bunch Area	0.0221/6	0.0457/6	eV s/N

At Booster injection, the voltage V_I required for particles with momentum p and charge Q to follow the nominal trajectory through the inflector is given by

$$eV_I = \frac{G}{R_I}\left(\frac{c^2p^2}{QE}\right). \tag{28}$$

Here $G = 0.017$ m is the gap between the cathode and septum of the inflector and $R_I = 8.74123$ m is the radius of curvature along the nominal trajectory. Using the tabulated values of cp and E at Booster injection, we obtain $V_I = 22.919$ kV.

The fractional momentum spread $\Delta p/p$ was measured by chopping a short notch out of the unbunched beam in the TTB line, and observing the turn-by-turn spreading of the notch in Booster at injection. This gives $\Delta p/p = \pm 3.9 \times 10^{-4}$. Observation of the notch also gives 15.1 μs for the revolution period at injection. The longitudinal emittance of the unbunched beam after accumulation in Booster is then 0.022 eV-s per nucleon.

Capture of injected beam in Booster occurs on a 6 ms porch at constant field. During this time the gap voltage is increased from 0 to 0.5 kV, capturing the beam into $h = 6$ stationary RF buckets. The tabulated bunch area at extraction was determined from measurements of the bunch width at extraction with $dB/dt = 80$ G/ms and $V_g = 30$ kV. The tabulated intensities (ions per bunch) were obtained during the FY 2007 RHIC run with 5.37×10^9 ions per pulse at the end of the Tandem to Booster (TTB) transfer line.

The six bunches are extracted from Booster in a single turn by means of a fast kicker and ejector septum magnet. Measurements of the beam width just downstream of the ejector give 95% horizontal and vertical emittances 4.2π and 2.8π (mm milliradians) respectively. After extraction, the ions pass through a stripper in the Booster to AGS (BTA) transport line where approximately 60% emerge in charge state +77. The stripper used this year consists of a 6.35 mg/cm^2 aluminum foil followed by a 8.48 mg/cm^2 "glassy" carbon foil mounted just downstream. The thicknesses have been optimized to produce the highest yield of Au^{77+}. The high uniformity of the glassy carbon, compared to that of the standard carbon stripper (23.1 mg/cm^2 graphite) used in the past, gives a significant reduction in the increase of longitudinal emittance due variable energy loss as the ions traverse the foil. With the standard carbon foil, this increase was approximately a factor of four; with the glassy carbon, the increase is a

factor of 1.8. The measured energy spread of the bunches in Booster at extraction is ± 18 MeV while that of the bunches in AGS at injection is ± 32 MeV. The measured average energy loss in the foils is 2.5 MeV per nucleon. This is significantly less the the 4 MeV per nucleon observed with the standard carbon stripper.

The Au^{77+} ions are injected into the AGS by means of a septum magnet and a fast kicker. Four batches of six bunches are injected at constant magnetic field to give a total of 24 bunches on the AGS injection porch.

The bunches are injected into stationary buckets at harmonic 24. Because of the reduced energy spread of ions emerging from the BTA stripper used this year, there is more than enough voltage available to match the buckets to the incoming bunches. (This was not possible with the standard carbon foil.) The required voltage was found to be approximately 100 kV per turn. Measurements of bunch width (55 ns) in the matched buckets give a six-bunch longitudinal emittance of 0.082 eV-s per nucleon. This is a factor of 1.8 greater than the emittance measured at Booster extraction. In addition to emittance growth due to variable energy loss as ions traverse the foil there is a phase mismatch caused by the average energy loss. Since the ions emerge from the foil with a smaller average velocity, the distance between bunch centers is reduced. (The time between bunch centers is unchanged.) This means that the 6 bunches of each batch entering the AGS will occupy slightly less than one fourth of the ring. The effect of the mismatch is to cause some dilution of longitudinal emittance during the merging process discussed below.

In the past, shortly after all four batches from Booster were injected, the harmonic 24 voltage was slowly reduced, adiabatically debunching the beam. Once debunched the beam was adiabatically rebunched into 4 bunches (in order to reach the bunch intensity desired for RHIC) and then accelerated to top energy at harmonic 12. Experience with this setup has shown that there can be beam instability and subsequent fast loss due to the small momentum spread of the unbunched beam. To avoid this, a new setup in which the 24 bunches are merged into 4 was developed and implemented this year. The merge is done in two steps. First the 24 bunches are merged into 12 by bringing on harmonic 12 while reducing harmonic 24. Then the 12 bunches are merged into 4 by bringing on harmonics 4 and 8 while reducing harmonic 12. This final merge is done with a single low-frequency. The resulting 4 equally spaced bunches are then accelerated to top energy at harmonic 12.

6 Gold Parameters in AGS

Parameter	Injection	Transition	Extraction	Unit
Q	77	77	77	
m	183.434144	183.434144	183.434144	GeV/c^2
W	0.098348276	6.9835334	8.8648782	GeV/N
cp	0.43911727	7.8597075	9.7516620	GeV/N
E	1.0294861	7.9146712	9.7960160	GeV/N
$B\rho$	3.7474454	67.075078	83.221098	Tm
β	0.42654027	0.99305547	0.99547224	
γ	1.1056216	8.5000	10.520480	
η	-0.804	0.0	0.00481	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	24	12	12	
hf	3.8025000	4.4264207	4.4370073	MHz
R	128.4526	128.4526	128.45798	m

Parameter	Injection	Extraction	Unit
V_g	101.4	179	kV
A_S	32.64	4895	eV s
dB/dt	0	0	G/ms
ϕ_s	0	180	degrees
F_s	4.039	0.0951	kHz
A_{bk}	32.64	4895	eV s
A_b	2.705	6×7.653	eV s
Δt	55.0	15.7	ns
ΔE	31.5	1863	MeV

Parameter	Injection	Extraction	Unit
No. of Bunches	24	4	
Bunch Spacing	262.985	676.131	ns
Ions/Bunch	1.70/6	1.68	10^9
Bunch Area	0.0824/6	0.233	eV s/N

Measurements of bunch width (16 ns) and gap volts per turn (180 kV) just before extraction give a single-bunch longitudinal emittance of 0.23 eV-s per nucleon.

7 Gold Parameters in RHIC

Parameter	Injection	Transition	Store	Unit
Q	79	79	79	
m	183.433122	183.433122	183.433122	GeV/c^2
W	8.8648288	20.382493	99.068867	GeV/N
cp	9.7516077	21.293276	99.995665	GeV/N
E	9.7959614	21.313625	100.000000	GeV/N
$B\rho$	81.1137824	177.117274	831.763013	Tm
β	0.99547224	0.99904526	0.99995665	
γ	10.520480	22.8900	107.396090	
η	-0.00713	0.0	0.00182	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	360	360	2520	
hf	28.023204	28.1237867	197.0461006	MHz
$2\pi R$	3833.845181	3833.845181	3833.845181	m

Parameter	Injection	Extraction	Unit
V_g	237.8	3000	kV
A_S	135.7	164.4	eV s
dB/dt	0	0	G/ms
ϕ_s	0	180	degrees
F_s	0.156	0.232	kHz
A_{bk}	135.7	164.4	eV s
A_b	45.9	137.8	eV s
Δt	15.7	4.0	ns
ΔE	1903	24048	MeV

Parameter	Injection	Store	Unit
No. of Bunches	60	60	
Bunch Spacing	214.108	213.148	ns
Ions/Bunch	1.20	1.14	10^9
Bunch Area	0.233	0.70	eV s/N

The four bunches are extracted from AGS one at a time by means of a fast kicker and thick ejector septum magnet. Prior to extraction, the AGS RF is synched to the RHIC RF and phase adjusted so that each AGS bunch will end up centered in the desired bucket on the RHIC injection porch.

After extraction, the bunches are transported down the AGS to RHIC (ATR) line to RHIC. Final stripping to charge state +79 occurs in a 45 mg/cm^2 tungsten foil in the line. The beam loss in the foil is less than 1%. In the past an aluminum oxide (Al_2O_3) flag was used for stripping. This gave a beam loss of 4% due to fragmentation of gold nuclei traversing the flag. Measurements of the normalized horizontal and vertical emittances in the ATR line give 10π (mm milliradians) in both planes. The ATR transport efficiency is close to 100%.

The measured bunch width at RHIC store after re-bucketing is approximately 4.0 ns. The gap voltage per turn is 3 MV.

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